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TRANSLATION

CALCULATING A THREE-DIMENSIONAL LAMINARY BOUNDARY LAYER ON SPREADING LINES

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Calculating a Three-Dimensional Laminary Boundary Layer on Spreading Lines

by

V.S.Avduyevskiy

For the case of a flow in the environs of spreading lines of a cone under an angle of attack , of an infinite cylinder with slipping and forward critical point, equations of a three-dimensional compressible boundary layer have been transformed into a system of ordinary differential equations .Introduced is a solving method,based on the use of integral ratios and a special form of approximating functions. Numerical solutions have been obtained in a wide range of parameter changes and formulas are given for the calculation of heat exchange , friction and boundary layer characteristics. Calculation results at partial parameter values are in satisfactory agreement with the numerical calculations of other authors.

1. Equations of laminary three-dimensional boundary layer in a compressible gas at stable flow along a curvilinear surface has the form of [1]:

Fig.1.

equations of motion

$$\frac{\rho u}{h_1}\frac{\partial u}{\partial x} + \frac{\rho w}{h_2}\frac{\partial u}{\partial z} + \rho v\frac{\partial u}{\partial y} + \frac{1}{h_1 h_2}\frac{\partial h_1}{\partial z}\rho u w - \frac{\rho w^2}{h_1 h_2}\frac{\partial h_2}{\partial x} = -\frac{1}{h_1}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \quad (1.1)$$

$$\frac{\rho u}{h_1}\frac{\partial w}{\partial x} + \rho w\frac{\partial w}{\partial z} + \rho v\frac{\partial w}{\partial y} - \frac{\rho u^2}{h_1 h_2}\frac{\partial h_1}{\partial z} + \frac{1}{h_1 h_2}\frac{\partial h_2}{\partial x}\rho u w = -\frac{1}{h_2}\frac{\partial p}{\partial z} + \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) \quad (1.2)$$

continuum equations...

$$\frac{1}{h_1}\frac{\partial \rho u}{\partial x} + \frac{1}{h_2}\frac{\partial \rho w}{\partial z} + \frac{\partial \rho v}{\partial y} + \frac{\rho u}{h_1 h_2}\frac{\partial h_2}{\partial x} + \frac{\rho w}{h_1 h_2}\frac{\partial h_1}{\partial z} = 0 \quad (1.3)$$

energy equation

$$\frac{\rho u}{h_1}c_p\frac{\partial T_0}{\partial x} + \frac{\rho w}{h_2}c_p\frac{\partial T_0}{\partial z} + \rho v c_p\frac{\partial T_0}{\partial y} = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T_0}{\partial y}\right) + \frac{\partial}{\partial y}\left[\frac{\mu}{P}(P-1)\frac{\partial}{\partial y}\frac{u^2+w^2}{2}\right] \quad (1.4)$$

x,z Here - curvilinear orthogonal coordinates on the surface; h_1, h_2 - Lame coefficients; y - coordinate,normal to the surface; u,w,v - projections of velocity vector

on the coordinate axes x,z,y (fig.1); p-static pressure; ρ - density; c_p - specific heat at constant pressure; λ - coefficient of heat conduction; μ -viscosity coefficient; T_0 - braking temperature. Prandtl number $P = \mu c_p / \lambda$.

2. If any one line z = const appears to be a geodetic line and the line of flow of an ideal fluid on the surface, then from condition

$$\frac{1}{h_1 h_2} \frac{\partial h_1}{\partial z} = 0 \quad (1.4a)$$

and from equation (1.2) follows a trivial solution $w \equiv 0, \partial p/\partial z = 0$.

In this way, everywheres along these lines, which we will call spreading lines, the components of the full velocity vector lie in one plane (but $\partial w/\partial z \neq 0$) just as at a two-dimensional flow. Branches of the flow outside of the boundary layer and within it diverge in both directions from the spreading line, and the boundary layer in the vicinity of these lines are calculated independently from the development of the boundary layer over the entire surface.

3. In many instances similarity transforms are possible, with the aid of which the system of boundary layer equations can be transformed into a system of ordinary differential equations We shall discuss the most important case of similarity, when a supersonic flow is directed around bodies.

a) Conical flow. Directing the line z = const along the formers of the cone, and lines x = const orthogonal to them we will obtain an expression for the Lame coefficient

$$h_1 = 1, \qquad h_2 = \varphi(z)\, x \qquad (3.1)$$

for the round cone with angle of semiopening θ_k

$$h_2 = R = \sin\theta_k x \quad (3.1a)$$

The spreading line correspond to the former of the cone; on it are maintained conditions:

$$\frac{\partial p}{\partial z} = \frac{\partial u_1}{\partial z} = 0, \quad w_1 = 0, \quad \frac{\partial w_1}{\partial z} = b \qquad (3.2)$$

Introducing variables

$$\eta = \sqrt{\frac{u_1}{\nu_w x} \frac{1}{2}} \int_0^y \frac{\rho}{\rho_w} dy, \quad f' = \frac{u}{u_1}, \quad g' = \frac{w}{u_1}, \quad S = \frac{T_0}{T_{01}} - 1 \quad (3.3)$$

we obtain from (1.1) - (1.4) a system of ordinary differential equations

$$\frac{1}{3}(lf'')' + (f + Kg)f'' = 0 \qquad \left(l = \frac{\rho\mu}{\rho_w\mu_w}\right) \tag{3.4}$$

$$\frac{1}{3}(lg'')' + (f + Kg)g'' - \frac{2}{3}fg' - K(g')^2 = -\frac{\rho_1}{\rho}\left(K + \frac{2}{3}\right) \tag{3.5}$$

$$\frac{1}{3}\left(\frac{l}{P}S'\right)' + (f + Kg)S' + \frac{u_1^2}{2T_w}\left[\frac{l}{P}(P-1)(f')^2\right]' = 0 \tag{3.6}$$

$$K = \frac{2}{3}\,\frac{\partial w_1/\partial z}{u_1\varphi} \tag{3.7}$$

The parameter K here characterizes the influence of three-dimensionality, index 1 corresponds to conditions outside of the layer and index w to the condition on the wall.

The ratio

$$\frac{\rho_1}{\rho} = \frac{T}{T_1} = \omega[1 - (f')^2] + S_w(1 - f')(1 + \omega) \tag{3.8}$$

where

$$\omega = \frac{u_1^2}{2J_1} = \frac{\varkappa - 1}{2}M_1^2 \tag{3.9}$$

Boundary conditions:

$$\begin{aligned} &f' = f = g' = g = 0, \quad S = S_w \quad \text{при } \eta = 0 \\ &f' \to 1, \quad g' \to 1, \quad S \to 0 \quad \text{при } \eta \to \infty \end{aligned} \tag{3.10}$$

b) Flow in the vicinity of a spreading line of a cylinder with slipping) Assuming gamma - angle of slipping (angle of sweepback for a delta wing). Directing lines z = const along the formers of the cylinder, we obtain $h_1 = h_2 = 1$. We will designate the rate of slipping $u_1 = a$ and will consider the case of flow $w_1 = bz^n$ in the vicinity of the spreading line ($w_1 \ll a$, ρ_1 = const, $T_1 \sim$ constant.

Having designated

$$\eta = \sqrt{\frac{n+1}{2}\,\frac{w_1}{\nu_w z}}\int_0^x \frac{\rho}{\rho_w}\,dy$$

$$f' = \frac{u}{u_1}, \quad g' = \frac{w}{w_1}, \quad S = \frac{T_0}{T_w} - 1, \quad \beta = \frac{2n}{n+1} \tag{3.11}$$

we obtain

$$(lf'')' + gf'' = 0 \tag{3.12}$$

$$(lg'')' + gg'' = \beta\left[(g')^2 - \frac{\rho_1}{\rho}\right] \tag{3.13}$$

$$\left(\frac{l}{P}S'\right)' + gS' + \frac{u_1^2}{2J_w}\left[\frac{l}{P}(P-1)(f')^2\right]' = 0 \tag{3.14}$$

at boundary conditions (3.10).

The system (3.4) - (3.6) for the case $\beta = 1$ can be obtained directly from (3.12) - (3.14) during transition to variables

$$f^{\circ} = f\sqrt{3/K}, \quad g^{\circ} = g\sqrt{3/K}, \quad \eta^{\circ} = \sqrt{3/K}\,\eta \qquad (3.14a)$$

and with the value K tending toward infinity.

For the case omega $\to 0$ equations (3.12) - (3.14) coincide with equations of the boundary layer at a transverse flow around a cylinder.

In case of greater w_1 values the transformation of similarity is possible, if $P = 1$, $\mu \sim 1/T$ with the use of Stewertson variables [4].

c) Three dimensional flow in the vicinity of forward critical point. We shall discuss the development of a boundary layer from the critical point along the line of spreading. We will plot a system of coordinates so that $h_1 = h_2 = 1$, which is absolutely true, if the investigated section of the surface can be turned into a plane.

If outside of the boundary layer omega ≈ 0, then the transformation of the similarity is possible at a condition $u_1 = ax^m$, $w_1 = bx^{m-1}z$ and by using variables

$$\eta = \sqrt{\frac{m+1}{2}\frac{u_1}{\nu_w x}} \int_0^y \frac{\rho}{\rho_w}\, dy, \qquad f' = \frac{u}{u_1}, \qquad g' = \frac{w}{w_1} \qquad (3.15)$$

In this case we obtain

$$(lf'')' + \left[f + (2-\beta)\frac{b}{a}g\right]f'' = \beta\left[(f')^2 - \frac{\rho_1}{\rho}\right] \qquad (3.16)$$

$$(lg'')' + \left[f + (2-\beta)\frac{b}{a}g\right]g'' - 2(\beta-1)f'g' = (2-\beta)\frac{b}{a}(g')^2 -$$
$$-\frac{\rho_1}{\rho}\left[(2-\beta)\frac{b}{a} + 2(\beta-1)\right] \qquad (3.17)$$

$$\left(\frac{l}{P}S'\right)' + \left[f + (2-\beta)\frac{b}{a}g\right]S' = 0 \qquad (3.18)$$

Boundary conditions ar determined (3.10). When $\beta \to 0$ the equations can be transformed into form (3.12) - (3.14) with the aid of substitutions

$$\frac{b}{a} = 1 + \frac{3}{2}K, \qquad \frac{b}{a}g = f + \frac{3}{2}Kg^{\circ} \qquad (3.19)$$

necessary for coordinating the selected coordinate systems on the surface. The case $b/a = 1$ corresponds to an axially-symmetrical flow, and equations (3.16) and (3.17) have a trivial solution $f \equiv g$. The case $b/a = 0$ correspond to a plain flow, and

equations (3.16) - (3.18) convert into (3.12) - (3.14). When $\beta \to 2$ the influence of three-dimensionality disappears and the boundary layer develops so, as in plain flow.

4. For approximated solutions it is convenient to use integral ratios, obtainable during the integration of the system (1.1)-(1.4) according to y from value y = 0 to the value , corresponding to the boundary of the layer. The obtained equations have the form of

$$\frac{\partial}{\partial x}(h_2\rho_1 u_1^2 \vartheta_{xx}) + \frac{\partial}{\partial z}(h_1\rho_1 u_1 w_1 \vartheta_{zx}) + \frac{\partial u_1}{\partial x} h_2\rho_1 u_1 \delta_x^* + \frac{\partial u_1}{\partial z} h_1\rho_1 w_1 \delta_z^* -$$

$$-\frac{\partial h_1}{\partial x}\rho_1 w_1^2 (\delta_z^* + \vartheta_{zz}) + \frac{\partial h_1}{\partial z}\rho_1 u_1 w_1 (\delta_z^* + \vartheta_{zx}) = h_1 h_2 \tau_{wx} \quad (4.1)$$

$$\frac{\partial}{\partial z}(h_1\rho_1 w_1^2 \vartheta_{zz}) + \frac{\partial}{\partial x}(h_2\rho_1 u_1 w_1 \vartheta_{xz}) + \frac{\partial w_1}{\partial z} h_1\rho_1 w_1 \delta_z^* + \frac{\partial w_1}{\partial x} h_2\rho_1 u_1 \delta_x^* -$$

$$-\frac{\partial h_1}{\partial z}\rho_1 u_1^2 (\delta_x^* + \vartheta_{xx}) + \frac{\partial h_2}{\partial x}\rho_1 u_1 w_1 (\delta_x^* + \vartheta_{xz}) = h_1 h_2 \tau_{wz} \quad (4.2)$$

$$\frac{\partial}{\partial x}[h_2\rho_1 u_1 c_p (T_0 - T_w)\,\vartheta_{xT}] + \frac{\partial}{\partial z}[h_1\rho_1 w_1 c_p (T_0 - T_w)\,\vartheta_{zT}] = h_1 h_2 q_w \quad (4.3)$$

Here q_w - specific thermal flow. τ_{wx} and τ_{wz} - stresses of component friction forces along the axes x and z

$$\vartheta_{xx} = \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{u}{u_1}\right) dy, \qquad \vartheta_{xz} = \int_0^\delta \frac{\rho u}{\rho_1 u_1}\left(1 - \frac{w}{w_1}\right) dy$$

$$\vartheta_{zz} = \int_0^\delta \frac{\rho w}{\rho_1 w_1}\left(1 - \frac{w}{w_1}\right) dy, \qquad \vartheta_{zx} = \int_0^\delta \frac{\rho w}{\rho_1 w_1}\left(1 - \frac{u}{u_1}\right) dy$$

$$\delta_x^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_1 u_1}\right) du, \qquad \delta_z^* = \int_0^\delta \left(1 - \frac{\rho w}{\rho_1 w_1}\right) dy$$

$$\vartheta_{xT} = \int_0^\delta \frac{\rho u}{\rho_1 u_1}\,\frac{T_{01} - T_0}{T_{01} - T_w}\, dy, \qquad \vartheta_{zT} = \int_0^\delta \frac{\rho w}{\rho_1 w_1}\,\frac{T_{01} - T_0}{T_{01} - T_w}\, dy \quad (4.4)$$

The sought for velocity and temperature distributions will be determined as a function of variable Y, representing the ratio of the variable of Dorodnitsyna y° to the parameter Θ, proportional to the thickness of the boundary layer

$$Y = \frac{y^\circ}{\Theta}, \quad y^\circ = \int_0^\delta \frac{\rho}{\rho_1} dy, \quad \Theta = \int_0^\delta \frac{T_0 - T_w}{T_{01} - T_w}\left(1 - \frac{T_0 - T_w}{T_{01} - T_w}\right)\frac{\rho}{\rho_1} dy \quad (4.5)$$

Let us now discuss velocity and temperature distribution (in relative coordinates)

$$\frac{T_0 - T_w}{T_{01} - T_w} = F_0'(Y), \quad \frac{u}{u_1} = F_0' + x_1 F_1'(Y), \quad \frac{w}{w_1} = F_0'(Y) + x_2 F_1'(Y) \quad (4.6)$$

The parameters x_1 and x_2 determining deformation of velocity profiles at a variable pressure from without the layer, should be found from three equations of the system (4.1) - (4.3). Function F_0' corresponds to temperature distribution and velocity in gradientless flow ($x_1=x_2=0$) and can be found from the solution of Blasius equations.

The form of the function F_1' is determined from the accurate solution of system (3.4) - (3.6) at $K = 0$ and $S_w = 0$, $P=1$. when the velocity distribution w/w_1 can be presented in form of

$$\frac{w}{w_1} = F_0'(Y) + (1 + \omega) F_1'(Y) \quad (4.6a)$$

In this way, the adopted velocity and temperature distribution assures total conformity with accurate solutions at a flow along the lines of spreading of a cone, where deformation of the profile w/w_1 appears to be maximum. We shall designate:

$$\int_0^\infty (1 - F_0') \, dY = A, \quad \int_0^\infty F_0' (1 - F_0') \, dY = B, \quad \int_0^\infty F' dY = D \quad (4.7)$$

$$\int_0^\infty F_1' F_0' dY = E, \quad \int_0^\infty (F'_1)^2 dY = H, \quad \int_0^\infty F_1' [1 - F_0'] \, dY = G = D - E$$

where for the given type of function F_0 and F_1:

$$A = 2.61, \ B = 1, \quad D = 1.163, \quad E = 0.652, \ H = 0.251, \quad G = 0.505 \ (4.7a)$$

All conditions of layer thickness can be further expressed through

$$\begin{aligned}
\vartheta_{xx} &= \Theta (1 + x_1 G - x_1 E - x_1^2 H) \\
\vartheta_{xz} &= \Theta (1 + x_1 G - x_2 E - x_1 x_2 H) \\
\vartheta_{xT} &= \Theta (1 + x_1 G), \\
\vartheta_{zz} &= \Theta (1 + x_2 G - x_2 E - x_2^2 H) \\
\vartheta_{zx} &= \Theta (1 + x_2 G - x_1 E - x_1 x_2 H) \\
\vartheta_{zT} &= \Theta (1 + x_2 G)
\end{aligned} \quad (4.7b)$$

$$\begin{aligned}
\delta_x^* &= \Theta \left[(1 + \omega) \frac{T_w}{T_0} A - (1 + \omega) x_1 D \right] + \omega \vartheta_{xx} \\
\delta_z^* &= \delta_x^* + D (x_1 - x_2) \Theta \\
\delta_z^* + \vartheta_{zx} &= \delta_x^* + \vartheta_{xz} \qquad (4.8)
\end{aligned}$$

The stresses of friction force components

$$\tau_{wx} = \frac{\mu_w w_1}{\Theta} (F_{0w}'' + x_1 F_{1w}''),$$

$$\tau_{wz} = \frac{\mu_w w_1}{\Theta} (F_{0w}'' + x_2 F_{1w}'') \qquad (4.9)$$

specific thermal flow

$$q_w = \frac{\lambda_w (T_0 - T_w)}{\theta} F''_{0w} \qquad (4.10)$$

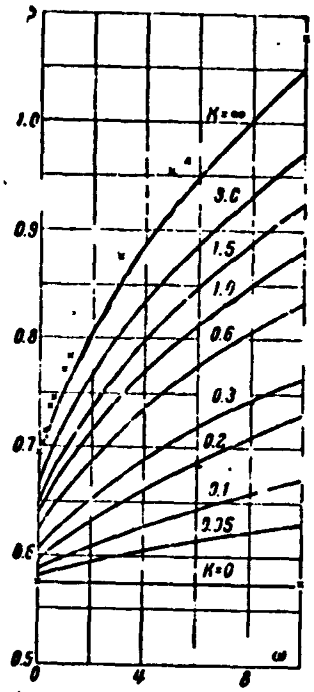

Fig.2.

At given functions F_0 and F_1 we have

insert eq.(4.10.a)...page 36

5. Using the method, described in previous paragraph, we will obtain a numerical solution of equations (3.4)-(3.6), (3.12)-(3.14) and (3.16) - (3.19). We will consider the case P = 1; $\mu \sim 1/T$.

a) Conical flow. The system of integral ratios has the form of :

$$\eta_{xx} + K\eta_{zx} = \frac{1}{3} f''_w = \frac{1}{3\eta_{TT}} (F''_{0w} + x_1 F''_{1w}) \qquad (5.1)$$

$$\eta_{xz} + 2K\eta_{zx} + \frac{2}{3}(\eta_{xz} + \eta_x^*) + K\eta_z^* = \frac{1}{3} g''_w = \frac{1}{3\eta_{TT}} (F''_{0w} + x_2 F''_{1w}) \qquad (5.2)$$

$$\eta_{xT} + K\eta_{zT} = -\frac{1}{3} \frac{S'_w}{S_w} \frac{F''_{0w}}{\eta_{TT}} \qquad (5.3)$$

where η is connected with δ by the ratio (3.11).

A comparison of (3.4) and (3.6) at P=1 shows, that $f''_w = S'_w/S_w$, and consequently

$$x_1 = 0, \quad \eta_{xx} = \eta_{TT} = r_{xT}, \quad \eta_{zT} = \eta_{zx} \qquad (5.3a)$$

In this way, to determine x_2 and η_{TT} remain equations (6.1) and (6.2).

Comparing (5.1) and (5.2) we obtain

$$\left(\frac{5}{3} + K\right)\frac{\eta_{xz}}{\eta_{TT}} + 2K\frac{\eta_{zz}}{\eta_{TT}} + \left(\frac{2}{3} + K\right)\frac{\eta_x^*}{\eta_{xz}} - K\frac{\eta_{zx}}{\eta_{TT}} = \left(1 + \frac{F''_{1w}}{F''_{0w}}\right)\left(1 + K\frac{\eta_{zx}}{\eta_{TT}}\right) \qquad (5.4)$$

Using the ratios (4.8) after transformations we obtain an equation for the determination of x_2

$$a x_2^2 + b x_2 + c(1 + \omega) = 0 \qquad (5.5)$$

$$a = K\left(G\frac{F''_{1w}}{F''_{0w}} + 2H\right) \qquad (5.3)$$ (5.8)...page 37

$$b = (K + 1)\frac{F''_{1w}}{F''_{0w}} + \left(3K + \frac{8}{3}\right)E \qquad (5.7)$$

$$c = -\left(K + \frac{2}{3}\right)\left(1 + A\frac{T_w}{T_\infty}\right) \qquad (5.8)$$

It can be seen easily, that at K = 0 in conformity with the accurate solution

$$x_2 = (1 + \omega) \quad (5.8a)$$

The sought for functions:

$$\frac{u}{u_1} = \frac{T_0 - T_w}{T_{01} - T_w} = F_0'(Y), \quad \frac{w}{w_1} = F_0'(Y) + x_2 F_1'(Y), \quad \eta = Y\eta_{rr} \quad (5.9)$$

$$\eta_{rr} = \frac{1}{3}\left[\frac{F''_{0w}}{1 + K(1 + x_2 G)}\right]^{\frac{3}{2}} \quad (5.10)$$

The values of velocity and temperature derivatives on the wall, necessary to calculate heat exchange and friction, equal

$$\left(\frac{S'_w}{S}\right) = f_w = \frac{F''_{0w}}{\eta_{rr}} = \sqrt{3} \cdot 0.47\,[1 + K(1 + x_2 G)]^{\frac{1}{3}} \quad (5.11)$$

$$g'' = f_w(1 + 1.29 x_2) \quad (5.12)$$

b) Flow in the vicinity of spreading lines of a cylinder with slipping. Integral equations have the form of

$$\eta_{zx} = \frac{F''_{0w}}{\eta_{rr}}, \quad \frac{3n+}{2}\eta_{zz} + n\eta_z' = \frac{n+1}{2}\frac{1}{\eta_{rr}}(F''_{0w} + x_2 F''_{1w}), \quad \eta_{zx} = \frac{F''_{0w}}{\eta_{rr}} \quad (5.13)$$

A comparison of (3.12) with (3.14) or (5.13) shows that

$$\eta_{rr} = \eta_{zx}, \quad \eta_{zx} = \eta_{rr} \quad (5.14)$$

After transformation we obtain

$$a_1 x_2^2 + b_1 x_2 + c_1(1 + \alpha) = 0, \quad a_1 = \left[\frac{F''_{1w}}{F''_{0w}} C + \frac{3n+1}{n+1} E\right]$$

$$b_1 = \frac{F''_{1x}}{F''_{0w}} + \frac{5n+1}{n+1} E, \quad c_1 = -\frac{2n}{n+1}\left[1 + A\frac{T_w}{T_\alpha}\right] \quad (5.15)$$

Having determined x_2, we obtain

$$\eta_{rr} = \left[\frac{F''_{0w}}{1 + Gx_2}\right]^{\frac{1}{3}} \quad (5.16)$$

Velocity and temperature distributions are determined by (5.9), the derivatives at the wall by formulae:

$$f_w = -\frac{S'_w}{S_w} = \frac{F''_{0w}}{\eta_{rr}} = 0.47\,(1 + x_2 G)^{\frac{1}{3}}, \quad g'_w = f_w(1 + 1.29 x_2) \quad (5.17)$$

c) A three dimensional flow in the vicinity of critical point. Integral ratios have the form of :

$$\frac{3m+1}{2}\eta_{xx}+\frac{b}{a}\eta_{zx}+m\eta_x^{\circ}=\frac{m+1}{2}\,\frac{1}{\eta_{TT}}\,[F_{0w}''+x_1F_{1w}''] \qquad (5.18)$$

$$\frac{3m-1}{2}\eta_{zz}+2\,\frac{b}{a}\,\eta_{xz}+\frac{b}{a}\,\eta_z^{\circ}+(m-1)\,\eta_x^{\circ}=\frac{m+1}{2}\,\frac{1}{\eta_{TT}}\,[F_{0w}''+x_2F_{1w}''] \qquad (5.19)$$

$$\frac{m+1}{2}\,\eta_{xT}+\frac{b}{a}\,\eta_{zT}=\frac{m+1}{2}\,\frac{F_{0w}''}{\eta_{TT}} \qquad (5.20)$$

The relationship between x_1 and x_2 will be obtained from condition at $\eta = 0$.

From equations (3.16) and (3.17) we have : (5.21)

$$f' = \frac{x_1F_{1w}'''}{\eta_{TT}^2} = -\frac{2m}{m+1}\,\frac{\rho_1}{\rho_w}, \qquad g' = \frac{x_2F_{1w}'''}{\eta_{TT}^2} = -\left(\frac{2}{1+m}\,\frac{b}{a}+2\,\frac{m-1}{m+1}\right)\frac{\rho_1}{\rho_w}$$

From expressions (5.21) we obtain:

$$\frac{x_2}{x_1}=\frac{b/a+m-1}{m} \qquad (5.22)$$

Comparing (5.18) and (5.20), we obtain:

See page 9a for equasions (5.23), (5.24), (5.25), and (5.26).

When $\beta \to 0$, $x_1 \to 0$ to determine x_2 it is necessary to make a change $x_1^{\circ} = x/\beta$; at $K_1 \to \infty$. $\beta \neq 2$, $x_1 \to 0$, making a change $x^{\circ} = K_1\,x_1/\beta$, we obtain an equation for x°. The value η_{TT} is determined from (5.20)

$$\eta_{TT}=\left[\frac{F_{0w}''}{(1+x_1C)+K_1(2-\beta)(1+x_2G)}\right]^{\frac{1}{2}} \qquad (5.27)$$

Velocity and temperature distribution is found by formula (5.9). Values of velocity and temperature distributions (derivatives) at the wall are equal:

$$-\frac{S_w'}{S_w}=\frac{F_{0w}'}{\eta_{TT}}, \qquad f_w''=\frac{F_{0w}''+x_1F_{1w}''}{\eta_{TT}}, \qquad g_w''=\frac{F_{0w}''+x_2F_{1w}''}{\eta_{TT}} \qquad (5.28)$$

6. Solutions of boundary layer equations obtained for the case $\mu \sim T$ and P=1 and during the use of these solutions to calculate heat exchange and friction it is necessary to introduce corrections . We will designate

$$N_w=\frac{\alpha L}{\lambda_w}, \qquad R_w=\frac{U\rho_wL}{\mu_w}, \qquad Cf_x=\frac{\tau_{wx}}{\rho_wU^2} \qquad (6.1)$$

where L and U- characteristic length and velocity. Expression for the thermal flow has the form of

$$q_w=\frac{N_w}{\sqrt{R_w}}\sqrt{\frac{\mu_w\rho_wU}{L}}\,\frac{c_{pw}(T_e-T_w)}{P}\,k_1k_2, \qquad T_e=\frac{T_{01}}{(1+\omega)}(1+\omega r) \quad (6.2)$$

$$a_2 x_1^2 + b_2 x_1 + c_2 = 0 \tag{5.23}$$

$$a = H[(\beta + 1)\beta + K_1(2\beta - 2) + (2 - \beta)^2 K_1^2 + \frac{F'_{1w}}{F'_{0w}} G[\beta + K_1(2\beta - 2) + (2 - \beta)^2 K_1^2] \tag{5.24}$$

$$b = \frac{F''_{1w}}{F''_{0w}}[\beta + K_1(2 - \beta)\beta] + E\left[\frac{4\beta + 2}{2}\beta + K_1(2 - \beta)\beta\right] \tag{5.25}$$

$$c = -\beta^2\left(1 + A\frac{T_w}{T_\infty}\right), \qquad \beta = \frac{2m}{m + 1} \tag{5.26}$$

Here r - coefficient of temperature restoration. Approximately $r \sim P^{n_1}$, where n_1 changes from 1/2 to 1/3; k_1-correcting multiple, taking into consideration the variability 1 and k_2 - multiple, considering the difference of P from one. Using the results of calculating on a flat plate with variable properties, we obtain approximately:

$$k_1 = \left(\frac{\mu^* \rho^*}{\mu_w \rho_w}\right)^{\frac{1}{3}} \left(\frac{\mu_1 \rho_1}{\mu_w \rho_w}\right)^{\gamma} \qquad \left(\gamma = \frac{1}{15}\frac{T_w}{T_{01}}\right) \quad (6.2a)$$

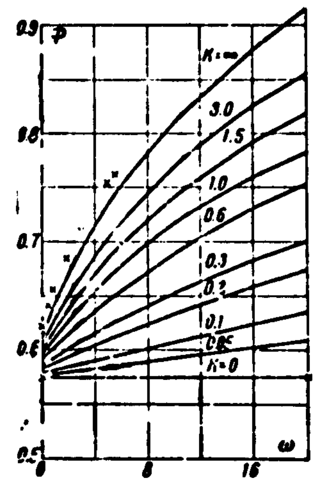

Fig.3

where μ^* and ρ^* have been determined at maximum temperature

$$T^* = T_1\left[\frac{T_w}{T_1} + \frac{1}{4\omega}\left(1 + \omega - \frac{T_w}{T_1}\right)^2\right]; \quad (6.2b)$$

$$T^* = T_1 \quad \text{при} \quad \omega \leqslant 1 - \frac{T_w}{T}$$

The value $k_2 = P^{n_2}$, where n_2 changes from 1/3 on the flat plate to ~ 0.45 in three dimensional flow at greater x_2. We will assume that $n \approx 0.4$.

To calculate friction we have analogously:

$$\tau_w = C f_w \sqrt{R_w} \sqrt{\frac{\mu_w \rho_w U^2}{L}}\, k_1 \quad (6.3)$$

a) Conical flow. In this case we have $L = x$, $U = u_1$ and

$$\frac{N_w}{\sqrt{R_w}} = \frac{1}{2} C f_{wx} R_w = \frac{1}{\sqrt{2}} f_w'' \qquad (6.4)$$

The dependence $\Theta = f_w''/(1+K)^{\frac{1}{2}}$ upon ω at various K and T_w/T_{01} is shown in fig.2 and 3; when K = 0 the solution coincides with the accurate ; when $K = \infty$ to compare points are shown data on the calculation of the job [4] .

Using formulas (5.17) it is easy to obtain basic characteristics of the boundary layer.

The angle between lines z = const and the lines of flow of an ideal liquid outside of the layer gamma = w_1/u_1 and the lines of flow at the wall $\gamma_w = \tau_{wz}/\tau_{wx}$; hence

$$\frac{\gamma_w}{\gamma} = \frac{g_w''}{f_w''} \qquad (6.5)$$

This ratio characterizes the effect of secondary flows. As is shown by calculations, g_{w*}/f_w rises at an increase in ω and decreases upon cooling the wall and increase in K. The expulsion thickness

$$\delta^* = \frac{\delta_x^* + K\delta_z^*}{1+K} \qquad (6.6)$$

Here the values δ_x^* and δ_z^* can be determined by formulas (4.8) and x_2 - from equations (5.5);

$$x_1 = 0, \qquad \Theta = \frac{\rho_w}{\rho_1}\eta_{TT}\sqrt{\frac{2\nu_w x}{u_1}} \qquad (6.6a)$$

the value η_{TT} is determined by formula (5.10).

b) Flow in the vicinity of spreading lines of a cylinder L = z, U = w_1 with slipping. In this case

$$\frac{N_w}{\sqrt{R_w}} = \frac{1}{2}C_{fwx}\sqrt{R_w} = \sqrt{\frac{\varkappa+1}{2}}f_w^* \qquad (6.7)$$

Next analogous to (6.5)

$$\frac{\gamma_w}{\gamma} = \frac{g_w^*}{f_w^*}, \qquad \gamma = \frac{w_1}{u_1} \qquad (6.8)$$

As in the case of flow on a cone, secondary flows rise at an increase in ω and β and decreases upon wall cooling.

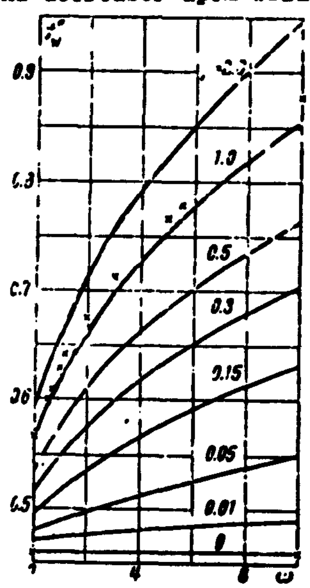

Fig.4

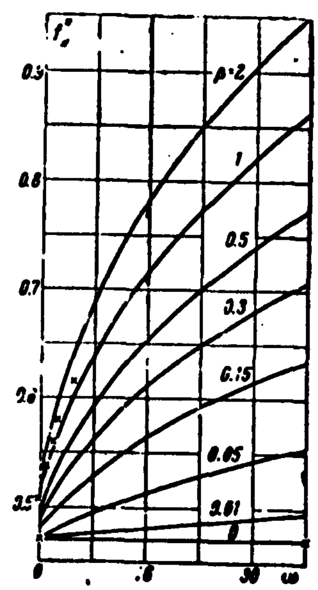

Fig.5

Calculation results are shown in fig.4 and 5. Given there are also data from [3] for β = 1, ω = (0-10) and T_w/T_{01} = (0.1) and from report [4] for ω = 0. The expulsion thickness $\delta^* = \delta_z^*$ is determined from equations (4.8), (5.15), (5.16).

c) Three dimensional flow in the vicinity of forward critical point. In this case

$$L = x, \quad U = u_1, \quad \frac{b}{a} = K_1 < 1, \quad \frac{N_w}{\sqrt{R_m}} = \sqrt{\frac{m+1}{2}}\left(-\frac{S_w'}{S_w}\right) \quad (6.9)$$

The results of calculating the dependence $(-S_w'/S_w)$ on K_1 and β are shown in fig.6 and 7. Plotted there are also data of calculations from report [4] for T_w/T_1 = 1 and data of report [2] for K_1=0 and K_1=1. The angle between the lines z=const and the direction of flow lines at the wall gamma$_w$ is determined by formula

$$\frac{\gamma_w}{\gamma} = \frac{g_w''}{f_w''}, \qquad \gamma = K_1 z \qquad (6.10)$$

As was shown by calculations , the effect of secondary flows decreases with the increase in β and disappears when β= 2; the value gamma$_w$/gamma = 1 at K_1=0 and K_1=1 and lower than unity, if $0 < K < 1$. It should be pointed out, that in this interval there is a zone, in which there is no solution of the system (3.16) - (3.18). Expulsion thickness

$$\delta^* = \frac{\delta_x^* + K_1\delta_z^*}{1+K_1} \qquad (6.11)$$

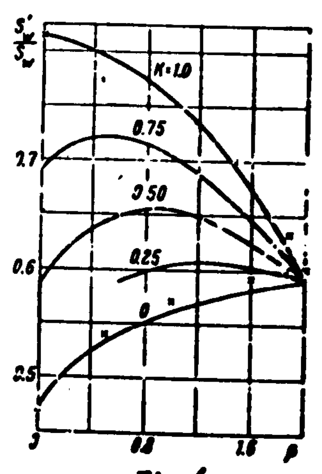

Fig.6

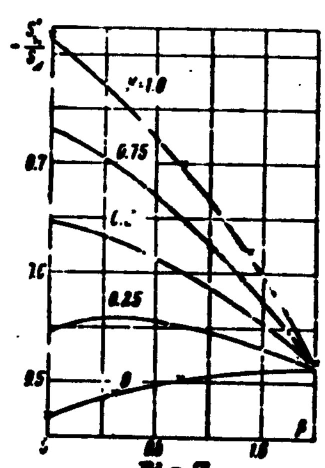

Fig.7

The values δ_x^* and δ_z^* are determined by formulas (4.8),(5.23),(5.27).

7. With the aid of the proposed method it is easy to obtain a solution of equations

of a two-and three-dimensional boundary layer in a wide range of parameter changes; As is evident from comparing with available numerical solutions, the accuracy of the method is perfectly sufficient for practical purposes.

In the report are given graphs for $T_w/T_{01} = 0$ and $T_w/T_{01} = 1$. Values for $0 < T_w/T_{01} < 1$ will be obtained with high degree by linear interpolation.

To calculate heat exchange under flight conditions it is necessary either to compute the distributions of parameters of an ideal liquid on the surface of a body, or use experimental data.

Submitted Sep. 14, 1961

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